The Network Topology of the Interbank Market *

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We provide an empirical analysis of the network structure of the Austrian interbank market based on a unique data set of the Oesterreichische Nationalbank (OeNB). We show that the contract size distribution follows a power law over more than 3 decades. By using a novel "dissimilarity" measure we find that the interbank network shows a community structure that exactly mirrors the regional and sectoral organization of the actual Austrian banking system. The degree distribution of the interbank network shows two different power law exponents which are one-to-one related to two sub-network structures, differing in the degree of hierarchical organization. The banking network moreover shares typical structural features known in numerous complex real world networks: a low clustering coefficient and a relatively short average shortest path length. These empirical findings are in marked contrast to interbank networks that have been analyzed in the theoretical economic and econo-physics literature.
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Over the past years the physics community has largely contributed to the analysis and to a functional understanding of the structure of complex real world networks. A key insight of this research has been the discovery of surprising structural similarities in very different networks, such as Internet, the World Wide Web, collaboration networks, biological networks, communication networks, electronic circuits and power grids, industrial networks, nets of software components and energy landscapes. See [1] for an overview. Remarkably, most of these real world networks show degree distributions which follow a power law, feature a certain pattern of cliquishness, quantified by the *clustering coefficient* of the network, and exhibit the so called *small world phenomenon*, meaning that the average shortest path between any two vertices ("degrees of separation") in the network can be surprisingly small [2]. Maybe one of the most important contributions to recent network theory is an interpretation of these network parameters with respect to stability, robustness, and efficiency of an underlying system, e.g. [3].

From this perspective financial networks seem a natural candidate to study. Indeed in the economic literature financial crises that have hit countries all around the globe have led to a boom of papers on banking-crises, financial risk-analysis and to numerous policy initiatives to improve financial stability. One of the major concerns in these debates is the danger of so called *systemic risk*: the large scale breakdown of financial intermediation due to domino effects of insolvency [4, 5]. The network of mutual credit relations between financial institutions is supposed to play a key role in the risk for contagious defaults. Some authors in the theoretical economic literature on contagion [6, 7, 8] suggest network topologies that might be interesting to look at. In [6] it is suggested to study a complete graph of mutual liabilities. The properties of a banking system with this structure is then compared to properties of systems with non complete networks. In [7] a circular graph is contrasted with a complete graph. In [8] a much richer set of different network structures is studied. Yet, surprisingly little is known about the *actual* empirical network topology of mutual credit relations between financial institutions. To our best knowledge the network topology of interbank markets has so far not been studied empirically.

The interbank network is characterized by the liability (or exposure) matrix L. The entries L_{ij} are the liabilities bank i has towards bank j. We use the convention to write liabilities in the rows of L. If the matrix is read column-wise (transposed matrix L^T) we see the claims or interbank assets, banks hold with each other. Note, that L is a square matrix but not necessarily symmetric. The diagonal of L is zero, i.e. no bank self-interaction exists. In the following we are looking for the bilateral liability matrix L of all (about $N = 900$) Austrian banks, the Central Bank (OeNB) and an aggregated foreign banking sector. Our data consists of 10 L matrices each representing liabilities for quarterly single month periods between 2000 and 2003. To obtain the Austrian interbank network from Central Bank data we draw upon two major sources: we exploit structural features of the Austrian bank balance sheet data base (MAUS) and the major loan register (GKE) in combination with an estimation technique.

*The views expressed in this paper are stritcly the views of the authors and do in no way commit the OeNB.

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The Austrian banking system has a sectoral organization due to historic reasons. Banks belong to one of seven sectors: savings banks (S), Raiffeisen (agricultural) banks (R), Volksbanken (VB), joint stock banks (JS), state mortgage banks (SM), housing construction savings and loan associations (HCL), and special purpose banks (SP). Banks have to break down their balance sheet reports on claims and liabilities with other banks according to the different banking sectors, Central Bank and foreign banks. This practice of reporting on balance interbank positions breaks the liability matrix L down to blocks of sub-matrices for the individual sectors. The savings banks and the Volksbanken sector are organized in a two tier structure with a sectoral head institution. The Raiffeisen sector is organized by a three tier structure, with a head institution for every federal state of Austria. The federal state head institutions have a central institution, Raiffeisenzentralbank (RZB) which is at the top of the Raiffeisen structure. Banks with a head institution have to disclose their positions with the head institution, which gives additional information on L. Since many banks in the system hold interbank liabilities only with their head institutions, one can pin down many entries in the L matrix exactly. This information is combined in a next step with the data from the major loans register of OeNB. This register contains all interbank loans above a threshold of 360 000 Euro. This information provides us with a set of constraints (inequalities) and zero restrictions for individual entries L_{ij}. Up to this point one can obtain about 90% of the L-matrix entries exactly.

For the rest we employ an estimation routine based on local entropy maximization, which has already been used to reconstruct unknown bilateral interbank exposures from aggregate information [9, 10]. The procedure finds a matrix that fulfills all the known constraints and treats all other parts (unknown entries in L) as contributing equally to the known row and column sums. These sums are known since the total claims to other banks have to reported to the Central Bank. The estimation problem can be set up as follows: Assume we have a total of K constraints. The column and row constraints take the form

$$\sum_{j=1}^{N} L_{ij} = b_i^r \quad \forall \quad i \quad \text{and} \quad \sum_{i=1}^{N} L_{ij} = b_j^c \quad \forall \quad j \qquad (1)$$

with r denoting *row* and c denoting *column*. Constraints imposed by the knowledge about particular entries in L_{ij} are given by

$$b^l \leq L_{ij} \leq b^u \quad \text{for some } i, j. \qquad (2)$$

The aim is to find the matrix L (among all the matrices fulfilling the constraints) that has the least discrepancy to some a priori matrix U with respect to the (generalized) cross entropy measure

$$\mathcal{C}(L, U) = \sum_{i=1}^{N} \sum_{j=1}^{N} L_{ij} \ln\left(\frac{L_{ij}}{U_{ij}}\right) \quad . \qquad (3)$$

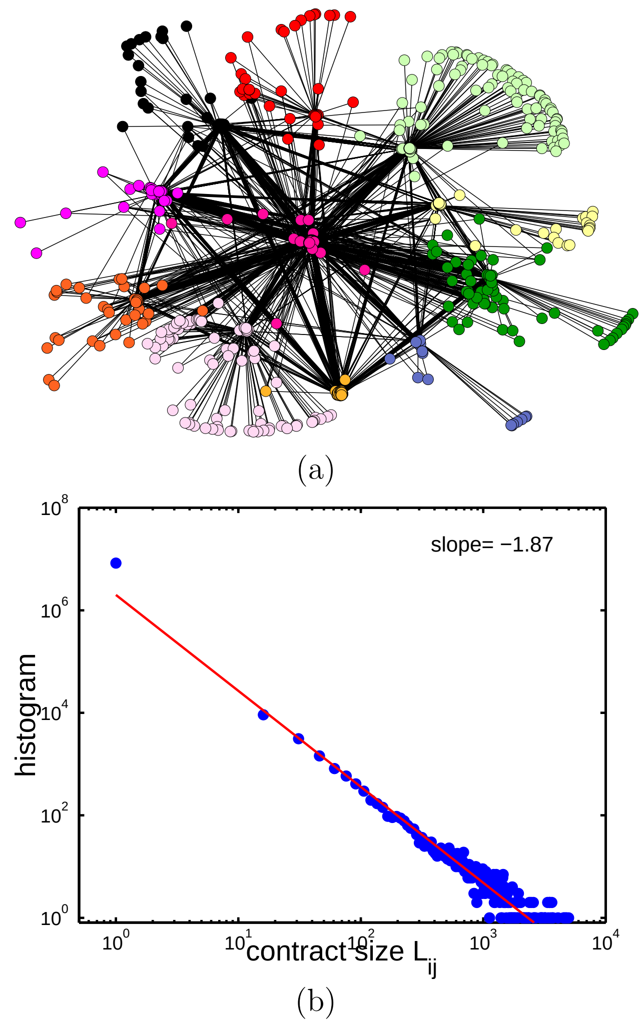

FIG. 1: The banking network of Austria (a). Clusters are grouped (colored) according to regional and sectorial organization: R-sector with its federal state sub-structure: RB yellow, RSt orange, light orange RK, gray RV, dark green RT, black RN, light green RO, light yellow RS. VB-sector: dark gray, S-sector: orange-brown, other: pink. Data is from the September 2002 L matrix, which is representative for all the other matrices. In (b) we show the contract size distribution within this network (histogram of all entries in L) which follows a power law with exponent -1.87. Data is aggregated from all 10 matrices.

U is the matrix which contains all known exact liability entries. For those entries (bank pairs) ij where we have no knowledge from Central Bank data, we set $U_{ij} = 1$. We use the convention that $L_{ij} = 0$ whenever $U_{ij} = 0$ and define $0 \ln(\frac{0}{0})$ to be 0. This is a standard convex optimization problem, the necessary optimality conditions can be solved efficiently by an algorithm described in [11, 12]. As a result we obtain a rather precise (see below) picture of the interbank relations at a particular point in time. Given L we plot the distribution (pdf) of its entries in Fig. 1(b). The distribution of liabilities follows a power law for more than three decades with an exponent of -1.87, which is within a range which is well known from wealth- or firm size distributions [13, 14].

To extract the network topology from these data, there are three possible approaches to describe the structure as a graph. The first approach is to look at the liability matrix as a *directed graph*. The vertices are all Austrian banks. The Central Bank OeNB, and the aggregate foreign banking sector are represented by a single vertex each. The set of all initial (starting) vertices is the set of banks with liabilities in the interbank market; the set of end vertices is the set of all banks that are claimants in the interbank market. Therefore, each bank that has liabilities with some other bank in the network is considered an initial vertex in the directed liability graph. Each bank for which this liability constitutes a claim, i.e. the bank acting as a counterparty, is considered an end vertex in the directed liability graph. We call this representation the *liability adjacency matrix* and denote it by A^l (^{l} indicating liability). $A^l_{ij} = 1$ whenever a connection starts from row node i and leads to column node j, and $A^l_{ij} = 0$ otherwise. If we take the transpose of A^l we get the interbank asset matrix $A^a = (A^l)^T$. A second way to look at the graph is to ignore directions and regard any two banks as connected if they have either a liability or a claim against each other. This representation results in an undirected graph whose corresponding adjacency matrix $A_{ij} = 1$ whenever we observe an interbank liability or claim. Our third graph representation is to define an undirected but weighted adjacency matrix $A^w_{ij} = L_{ij} + L_{ji}$, which measures the gross interbank interaction, i.e. the total volume of liabilities and assets for each node. Which representation to use depends on the questions addressed to the network. For statistical descriptions of the network structure matrices A, A^a, and A^l will be sufficient, to reconstruct the community structure from a graph, the weighted adjacency matrix A^w will be the more useful choice.

There exist various ways to find functional clusters within a given network. Many algorithms take into account local information around a given vertex, such as the number of nearest neighbors shared with other vertices, number of paths to other vertices, see e.g. [15, 16]. Recently a global algorithm was suggested which extends the concept of vertex betweenness [17] to links [18]. This elegant algorithm outperforms most traditional approaches in terms of mis-specifications of vertices to clusters, however it does not provide a measure for the differences of clusters. In [19] an algorithm was introduced which - while having at least the same performance rates as [18] - provides such a measure, the so-called dissimilarity index. The algorithm is based on a distance definition presented in [20].

For analyzing our interbank network we apply this latter algorithm to the weighted adjacency matrix A^w. As the only preprocessing step we clip all entries in A^w above a level of 300 m Euro for numerical reasons, i.e. $A^w_{\text{clip}} = \min(A^w, 300\text{m})$. The community structure obtained in this way (Fig. 1a) can be compared to the actual community structure in the real world. The result for the community structure obtained from one representative data set is shown in Fig. 2. Results from other datasets are practically identical. The algorithm identifies communities of banks which are coupled by a two or three tier structure, i.e. the R, VB, and S sectors. For banks which in reality are not structured in a hierarchical way, such as banks in the SP, JS, SM, HCL sectors, no strong community structure is expected. By the algorithm these banks are grouped together in a cluster called 'other'. The Raiffeisen sector, with its substructure in federal states, is further grouped into clusters which are clearly identified as R banks within one of the eight federal states (B,St,K,V,T,N,O,S). In Fig. 2 these clusters are marked as e.g. 'RS', 'R' indicating the Raiffeisen sector, and 'S' the state of Salzburg. Overall, there were 31 mis-specifications into wrong clusters within the total $N = 883$ banks, which is a mis-specification rate of 3.5 %, demonstrating the quality of the dissimilarity algorithm and - more importantly - the quality of the entropy approach to reconstruct matrix L.

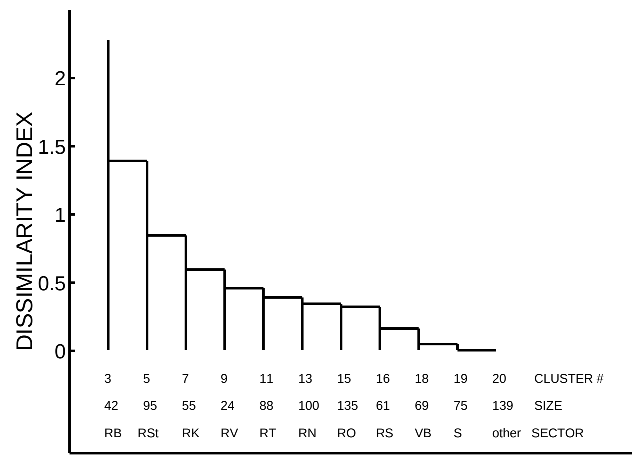

FIG. 2: Community structure of the Austrian interbank market network from the September 2002 data. The dissimilarity index is a measure of the "differentness" of the clusters.

Degree Distributions: Like many real world networks, the degree distribution of the interbank market follow power laws for all three graphs A^l, A^a, and A. Figure 3 (a) and (b) show the out-degree (liabilities) and in-degree (assets) distribution of the vertices in the interbank liability network. Figure 3 (c) shows degree distribution of the interbank connection graph A. In all three cases we find two regions which can be fitted by a power. Accordingly, we fit one regression line to the small degree distribution and one to the obvious power tails of the data using an iteratively re-weighted least square algorithm. The power decay exponents γ_{tail} to the tails of the degree distributions are $\gamma_{tail}(A^l) = 3.11$, $\gamma_{tail}(A^a) = 1.73$, and $\gamma_{tail}(A) = 2.01$. The size of the out degree exponent is within the range of several other complex networks, like e.g. collaboration networks of actors (3.1) [21], sexual contacts (3.4) [22]. Exponents in the range of 2 are for example the Web (2.1) [23] or mathematicians' collaboration networks (2.1) [24], and examples for exponents of about 1.5 are email networks (1.5) [25] and co-autorships (1.2) [26]. For the left part of the distribution (small degrees) we find $\gamma_{small}(A^l) = 0.69$, $\gamma_{small}(A^a) = 1.01$, and $\gamma_{small}(A) = 0.62$. These exponents are small compared to other real world networks. Compare e.g. with foodwebs (1) [27]. We have checked that the distribution for the low degrees is almost entirely dominated by banks of the R sector. Typically in the R sector most small agricultural banks have links to their federal state head institution and very few contacts with other banks, leading to a strong hierarchical structure, which is also visible by plain eye in Fig. 1a. This hierarchical structure is perfectly reflected by the small scaling exponents [28].

Clustering Coefficient: To quantify clustering phenomena within the banking network we use the so-called clustering coefficient C, defined by

$$C = \frac{3 \times (\text{number of triangles on the graph})}{\text{number of connected triples of vertices}} \quad . \qquad (4)$$

It provides the probability that two vertices that are connected to any given vertex are also connected with one another. A high clustering coefficient means that two banks that have interbank connections with a third bank, have a greater probability to have interbank connections with one another, than will any two banks randomly chosen on the network. The clustering coefficient is only well defined in undirected graphs. We find the clustering coefficient of the connection network (A) to be $C = 0.12 \pm 0.01$ (mean and standard deviation over the 10 data sets) which is relatively small compared to other networks. In the context of the interbank market a small C is a reasonable result. While banks might be interested in some diversification of interbank links keeping a link is also costly. So if for instance two small banks have a link with their head institution there is no reason for them to additionally open a link among themselves.

Average Shortest Path Length: We calculate the average path length for the three networks A^l, A^a, A with the Dijkstra algorithm [29] and find an average path length of $\bar{\ell}(A^l) = \bar{\ell}(A^a) = 2.59 \pm 0.02$. Note the possibility that in a directed graph not all nodes can be reached and we restrict our statistics to the giant components of the directed graphs. The average path length in the (undirected) interbank connection network A is $\bar{\ell}(A) = 2.26 \pm 0.03$. From these results the Austrian interbank network looks like a very small world with about three degrees of separation. This result looks natural in the light of the community structure described earlier. The two and three tier organization with head institutions and sub-institutions apparently leads to short interbank distances via the upper tier of the banking system and thus to a low degree of separation.

Our analysis provides a first picture of the interbank

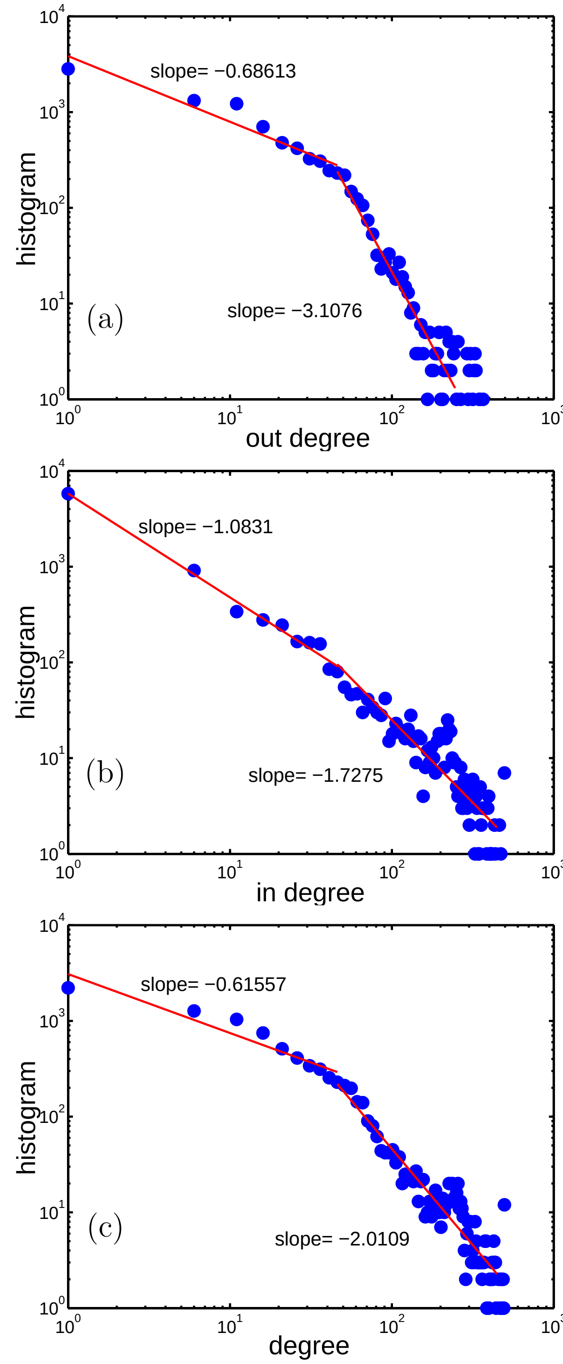

FIG. 3: Empirical out-degree (a) in-degree and (b) distribution of the interbank liability network. In (c) the degree distribution of the interbank connection network is shown. All the plots are histograms of aggregated data from all the 10 datasets.

network topology by studying a unique dataset for the Austrian interbank market. Even though it is small the Austrian interbank market is structurally very similar to the interbank system in many European countries including the large economies of Germany, France, and Italy. We show that the liability (contract) size distribution follows a power law, which can be understood as being driven by underlying size and wealth distributions of the banks, which show similar power exponents. We find that the interbank network shows – like many other realistic networks – power law dependencies in the degree distributions. We could show that different scaling exponents relate to different network structure in different banking sectors within the total network. The scaling exponents by the agricultural banks (R) are very low, due to the hierarchical structure of this sector, while the other banks lead to scaling exponents also found in other complex real world networks. Regardless of the size of the scaling exponent, the existence of a power law is a strong indication for a stable network with respect to random bank defaults or even intentional attack [3]. The interbank network shows a low clustering coefficient, a result that mirrors the analysis of community structure which shows a clear network pattern, where banks would first have links with their head institution, whereas these few head institutions hold links among each other. A consequence of this structure is that the interbank network is a small world with a very low "degree of separation" between any two nodes in the system. A further important message of this work is that it allows to exclude large classes of unrealistic types of networks for future modeling of interbank relations, which have so far been used in the literature.

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